Evolution of the mode of convection within terrestrial planets

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Abstract. Magma oceans, plate tectonics, and stagnant-lid convection have transferred heat out of the terrestrial planets at various times in their histories. The implications of the existence of multiple branches are graphically illustrated by approximating the globally averaged mantle heat flow as a function of the interior potential temperature. For this assumption to be valid, the mantle heat flow needs to be able to change rapidly relative to the potential temperature, or, equivalently, lithosphere needs to be a small fraction of the mass planet. This criterion is satisfied by the Earth, Venus, and Mars, but not the Moon. At a given potential temperature the function may be multivalued with a separate branch representing each mode of convection. The heat flow evolves along a branch as the potential temperature changes depending on whether the heat flow is greater or less than the global radioactive heat generation. When the end of a branch is reached, the state of the system jumps to another branch, quickly changing the global heat flow. Examples include transitions from a magma ocean to plate tectonics, probably on the Earth and Mars, and conceivably Venus; and the transition from a stagnant-lid planet to a magma ocean on Venus and the eventual return to a stagnant-lid planet.

1. Introduction

Current evidence indicates that the terrestrial planets accreted hot with the silicate interiors at least partly molten. Rock samples are available from Mars, the Moon, and the Earth. The moon-forming impact left both the Moon and the Earth partly vaporized [Cameron, 1997]. Hard evidence for this event around 4.45 Ga comes mainly from the Moon. An initial magma ocean created the buoyant highland crust of the Moon, which remains in place except where it has been blasted away by impacts [e.g., Warren, 1993]. Rock evidence relating to the early Earth is more tenuous, as the subsequent history of the Earth has been very eventful. Daughter products of extinct and extant isotopes indicate that the core and the silicate mantle last separated around 4.45 Ga, which is typically taken as the time of the moon-forming impact. In addition, both radiogenic and primordial Xe were depleted from the Earth relative to other rare gases at about that time [Ozima and Podosek, 1999]. This indicates long-lasting conditions favorable to Xe escape following the moon-forming impact, as mere ballistic ejection of gas during the impact did not select for Xe, but the Xe escape cannot now be quantified in terms of the state of the surface and the atmosphere.

Mars was unaffected by the moon-forming impact but also accreted hot. The great age and low Pt-group elements in the Martian meteorite ALH-84001 imply that the core of Mars formed before 4.5 Ga and that the interior of the planet was hot then [Kong et al., 1999; Warren et al., 1999]. Hints of the existence of a magma ocean are found in the isotope systematics of Martian meteorites [Blichert-Toft et al., 1999]. The possibility of Martian plate tectonics in the southern Highlands has been discussed by Connerney et al. [1999] on the basis of magnetic lineations and in the northern lowlands by Sleep [1994] on the basis of geomorphology.

Surface morphology and cratering ages indicate that plate tectonics does not occur now on Venus. However, analogy to the Earth and possibly Mars indicates that plate tectonics may have occurred in the past [Turcotte, 1993; Turcotte et al., 1999]. Combined, this evidence indicates that the mode of global convection within the terrestrial planets has changed with time. No magma oceans currently exist on the inner planets, and plate tectonics are active only on the Earth. The purpose of this paper is to explore possible planetary thermal histories where the mode of convection changes suddenly or gradually over time.

2. Global Thermal Histories

I discuss the mode of global convection and its efficacy by using two parameters, the potential temperature of the adiabatic planetary interior \( T \) and the globally averaged heat flow escaping from the mantle \( q \). Conservation of energy then implies that

\[
\frac{4\pi r_p^3}{3} \rho C \frac{dT}{dt} = -4\pi r_p^2q + \frac{4\pi r_m^3}{3} \rho A ,
\]

where \( r_p \) is planetary radius, \( t \) is time, \( \rho \) is density, \( C \) is heat capacity per mass, and \( A \) is radioactive heat generation per mass. (Heat flow generated by radioactive elements in the crust does not significantly affect interior convection except for removing a possible heat source from the mantle. The existence of the core and mantle plumes is ignored for the time being.) Alternatively, it is convenient to express the heat contribution from radioactivity as an equivalent heat flow \( q_r \) at the surface:

\[
\frac{4\pi r_p^3}{3} \rho C \frac{dT}{dt} = -4\pi r_p^2q + 4\pi r_p^2q_r .
\]
2.1. Heat Flow as Function of Potential Temperature

To the extent that the silicate interior behaves like a simple viscous fluid, higher interior temperatures are expected to imply lower viscosities, greater lateral temperature gradients, and hence more vigorous convection. Heat is conducted through an upper thermal boundary layer which may be either mobile or stagnant. If the mass of the thermal boundary layer is small enough compared to the mass of the planet, the global surface heat flow may change much more rapidly than the interior potential temperature does. In that case, it is convenient to represent the interior potential temperature $T$ as a continuously varying function of time and the global heat flow $q(T)$, an instantaneous function of $T$.

To represent heat flow as $q(T)$, I have implicitly assumed that the planet is large compared to the thickness of the lithosphere in two ways. First, in the heat balance in (1) and (2), it is assumed that the potential temperature changes throughout the planet. This assumption is justifiable when the cool lithosphere makes up a small fraction of the planet’s mass. A related criterion is that the lithosphere is thin enough that subduction of the global area of lithosphere does not drastically cool the interior of the planet, as is the case for Venus [Turcotte et al., 1999].

The second large planet assumption is that the time for the lithosphere to adjust to the interior conditions within the planet is small compared with the time that it takes radioactivity to heat up the interior. The effective timescale for the lithosphere is short for plate tectonics and magma oceans, where much of the heat loss is near upwellings, like ridge axes. Quasi-steady state is reached in the time it takes seafloor spreading or magma ocean processes to resurface the area of the planet. For the Earth this time $t_p$ is 170 m.y., the area of the planet, $5.1 \times 10^8 \text{km}^2$, divided by the global rate of seafloor spreading, $3 \text{km}^2 \text{yr}^{-1}$. (Alternatively, 100 m.y., the time it takes seafloor spreading to resurface the ocean basins, can be used.) For stagnant-lid convection the conductive timescale of the lithosphere, which may potentially be quite large, is relevant. I examine this condition for stagnant-lid convection in section 2.4.3.

2.2. Heat Balance

A particular mode of convection may have a global heat flow which exceeds the interior radioactive heat generation $q_r$ so that the planet cools with time or a smaller global heat flow which allows radioactivity to heat up the planet. I briefly review evidence for the radioactivity and cooling terms in (1) and (2) to put these possibilities in context.

2.2.1. Radioactive heat generation. The probable amount of heat flow due to radioactive heat generation on the other terrestrial planets $q_r$ in (2) is obtained by scaling from the Earth. To do this, I make the assumption that the radioactivity per mass of the other terrestrial planets does not differ drastically from that of the well-sampled Earth.

An upper limit of the current mantle radioactivity heat generation is given by the observed terrestrial mantle heat flow of about 80 mW m$^{-2}$ by applying (2), as it is generally believed that the mantle is now cooling. More sophisticated estimates yield that radioactive heat generation supplies about half of the average surface heat flux from the mantle [e.g., McKenzie and Richter, 1981]. This yields a range for $q_r$ of 40–80 mW m$^{-2}$.

An upper estimate for $q_r$ on the early Earth is obtained by adjusting for radioactive decay and for the accumulation of radioactive elements in the crust over time. The radioactivity heat generation was about a factor of 4 greater than present in the Hadean [Van Schmus, 1995]. Summing the upper limit of 80 mW m$^{-2}$ for $q_r$ with the radioactive heat generation within the continents, 20 mW m$^{-2}$ averaged globally, yields 100 mW m$^{-2}$. Adjusting for radioactive decay yields an upper limit of $q_r$ in the Hadean of 400 mW m$^{-2}$. A lower estimate is obtained by adjusting the lower current estimate of 40 mW m$^{-2}$ for radioactive decay to obtain 160 mW m$^{-2}$.

Scaling is done to the other planets by assuming that they have a similar composition to the Earth. It is convenient to scale to the acceleration of gravity $g$, which is known for the other inner planets. The inverse square law for gravity (or, equivalently, Gauss’ law),

$$g = \frac{GM}{r_p^2} = \frac{4\pi GM}{4\pi r_p^2} = \frac{4\pi G \rho r_p^3}{3 r_p^3},$$

where $G$ is the gravitation constant, implies that $g$ is proportional to planetary mass divided by its surface area. Similarly, the heat flow owing to radioactivity is

$$q_r = \frac{AM}{4\pi r_p^2} = \frac{A \rho r_p^3}{3 r_p^3}.$$

This yields estimates for $q_r$ for Venus, Mars, and the Moon of 90, 38, and 16.5%, respectively, of that of the Earth. Scaling the Hadean upper estimate for the Earth gives 360, 150, and 66 mW m$^{-2}$ for Venus, Mars, and the Moon, respectively. Scaling the current estimate range for the Earth of 40–80 mW m$^{-2}$ yields 36-72, 15-30, and 6.6–13.2 mW m$^{-2}$, respectively.

2.2.2. Transient cooling. The amount of heat liberated by transient cooling of a planet during its history depends on its specific heat and on the total transient cooling that can take place. The Earth again provides a useful reference point from which to begin scaling. I now consider the core explicitly.

About 1/3 of the Earth’s mass is in its core, and the remainder is in its crust and mantle. The specific heat per mass of the silicate is about twice that of the iron, implying that 4/5 of the specific heat is in the mantle and 1/5 is in the core. Taking this into account, the specific heat of the entire Earth (mantle plus core) is around $1040 \text{ Jkg}^{-1} \text{K}^{-1}$ [Stacey and Loper, 1984], which implies that the specific heat of the entire body is $6.25 \times 10^{17} \text{ JK}^{-1}$. From (1) a heat flow of 1 W m$^{-2}$ cools the Earth at a rate of 2.57 K per million years. No gross error in this number results if the core cools at a different rate than the mantle, but the cooling of the core must be explicitly considered if one wishes to discuss mantle plumes arising from the basal thermal boundary layer or the existence of a magnetic field [Stacey and Loper, 1984; Sleep et al., 1988; Davies, 1993a; Nimmo and Stevenson, 2000].

The rate that at planetary interior heats up by radioactivity in the absence of surface heat flow is, from (1), independent of its size

$$\frac{\partial T}{\partial t} = \frac{A}{C}.$$  

A probable range for this rate is obtained from the above examples involving the Earth. At an early time when the interior radioactive heat generation was equivalent to a global heat flow of 400 mW m$^{-2}$, the interior would warm up by 1 K per million years. At present, with an equivalent heat flow from radioactivity of 40 mW m$^{-2}$, it would warm up by 0.1 K per million years. These are increased by a factor of 5/4 if only the mantle
and not the core warmed up, that is, to 1.25 and 0.125 K per million years, respectively.

The cooling rate at a given global heat flow on the other planets scales inversely proportional to the acceleration of gravity, as does \( q_e \) in (4). That is, the globally averaged heat flow owing to cooling is from (1)

\[
q_e = -\frac{C \partial T}{\partial t} \frac{M}{4 \pi r^2}.
\]

That is, a global heat flow of \( 1 \text{ W m}^{-2} \) cools Venus, Mars, and the Moon at a rate of 2.86, 6.76, and 15.58 K per million years, respectively.

The amount of interior heat available to drive convection during a planet's life is limited by the temperature range between when the planet first formed a solid lid (that is, a lithosphere) and the present interior temperature. For the Earth, vast amounts of heat were lost following the moon-forming impact by radiative heat transfer from the top of a rock vapor atmosphere and later from a fully molten surface. The epochs lasted only thousands of years [Zahnle and Sleep, 1996]. A solid lid formed when the mantle adiabat was "900 K hotter than the current mantle adiabat [Abe, 1997]. The Archean adiabat was "150 K higher than the current one [Abbott et al., 1994]. The available heat is somewhat less on smaller planets, where the melting point does not increase greatly with depth, but the statement that only the heat from radioactivity and from cooling of several hundred kelvins has been available to provide heat flow still holds.

2.3. Modes of Convection

Magma oceans, plate tectonics, and stagnant-lid convection have occurred in the terrestrial planets. It is convenient to discuss them in the reverse order that they occurred on an initially hot planet because the physics of stagnant-lid convection and plate tectonics are better understood than those of magma oceans.

2.3.1. Stagnant-lid convection. An essentially stagnant lid develops on top of a convecting interior within a fluid with a strongly temperature dependent viscosity [Solomatov, 1995]. Geologically, this results in a one-plate planet where convection driven by a boundary layer at the base of the lithosphere extends downward throughout the adiabatic interior. This is currently the main mode of heat transfer on Mars, Venus, and the Moon. On the Earth it occurs as secondary convection beneath the plates and transfers moderate amounts of heat to the oceanic lithosphere [Davaille and Jaupart, 1994] and to the cratons [Doin et al., 1997; Jaupart et al., 1998; Dumoulin et al., 1999].

The scaling relationships of stagnant-lid convection are well constrained. For a linear fluid the quasi-steady heat flow from free convection at the base of the lithosphere, from Davaille and Jaupart [1993a, b], is

\[
q_f = 0.47 kT_\text{L} \left[ \frac{\rho g \alpha T_n}{\kappa \eta_0} \right]^{1/3},
\]

where \( k \) is thermal conductivity, \( \rho \) is density, \( g \) is the acceleration of gravity, \( \alpha \) is the thermal expansion coefficient, \( \kappa = k/\rho C \) is thermal diffusivity, \( \eta_0 \) is the half-space viscosity, \( T_n \) is the temperature to change viscosity by a factor of \( e \), and \( \rho C \) is specific heat per volume. The temperature contrast across the boundary layer is \( 2 T_n \) to \( 3 T_n \).

The scaling can be extended to a nonlinear viscosity [Solomatov, 1995; Solomatov and Moresi, 1997; Reese et al., 1998, 1999; Kawada and Honda, 1999]. Crudely, nonlinear fluid is nearly equivalent to a linear fluid with an apparent \( T_n \) a factor of \( n \) larger than the actual \( T_n \). More precise parameterizations are complicated and are not repeated here.

2.3.2. Plate tectonics. Plate tectonics differs from stagnant-lid convection in that the entire temperature difference of the sinking slab from the adiabatic mantle, not just the temperature scale \( T_n \), drives convection. Plate boundaries need to act as weak zones for this process to function.

A widely applied kinematic scaling based on treating the cooling lithosphere as a half-space has been obtained from the observed depth-age relationship at ridges. This behavior occurs to a good approximation in numerical simulations which show plate-like behavior. Locally, the heat flow is inversely proportional to the square root of the spreading rate. Globally, it is inversely proportional to the square root of the lifetime of oceanic crust,

\[
q_p = \left[ \frac{T_L}{T_{\text{L0}}} \right] \frac{1}{\sqrt{\eta_p}}
\]

where the heat flow is in \( \text{W m}^{-2} \), the lifetime is in millions of years, \( T_L \) is the temperature contrast across the lithosphere, \( T_{\text{L0}} \) is the current temperature contrast for the Earth, and \( \eta_p \) is the lifetime of oceanic lithosphere. Currently, the lifetime is 100 m.y., the area of ocean basins, \( 3 \times 10^8 \text{ km}^2 \), divided by the production rate of new seafloor area per time, \( 3 \text{ km yr}^{-1} \). At an early time before significant continents existed, the entire surface area, \( 5.1 \times 10^8 \text{ km}^2 \), of the Earth participated in spreading and subduction.

It has been conjectured for some time that a heat flow scaling relationship of the form of (7),

\[
q_{\text{pl}} = C_{\text{pt}} kT_L \left[ \frac{\rho g \alpha T_n}{\kappa \eta_0} \right]^{1/3},
\]

where \( C_{\text{pt}} \) is a constant and \( T_L \) is the temperature contrast across the lithosphere, also applies to planets [ Olson and Corcos, 1980; Solomatov, 1995]. Moresi and Solomatov [1998] show that the relationship does apply when plates are represented as having a simple yield stress rheology. This result may very well apply as a global average, but terrestrial plate velocities vary over at least a factor of 5, indicating that the effect of lithosphere may be more complicated than just a yield stress. Equivalently, the average heat flow from (7) for individual plates from ridge to either passive margin or trench varies by more than a factor of 2.

Tackley [1998] and Bercovici [1998] represent plate boundaries by a damage rheology where at steady state the shear stress increases with strain rate at low strain rates but decreases with strain rate at high strain rates. This allows fault zones, once established, to remain weak, as would be expected from the geological observation that fault zones are often reactivated after periods of quiescence. These models, the models by Tackley [2000], and the models on Tackley's unpublished Web sites show plate-like behavior. Some details of modern plate motion, including transform faults and one-sided subduction, do not arise in the simulations. This work is significant progress in quantitatively predicting plate behavior but not yet to the point that good predictions for the early Earth and for other planets are obtained.

This being the case, I adopt a phenomenological approach beginning with the surface heat flow to illustrate properties of a successful parameterization for situations where (8) appears
suspect. Partial melting of ascending material at ridge axes limits the mantle potential temperature range over which plate tectonics can occur. Ascending melts serve to unpin spreading at ridge axes by dike intrusion and by forming easily deformed magma chambers. This effect is represented in Tackley’s unpublished models.) At sufficiently low spreading rates or low melt potential temperatures, little or no melt is produced, and unpinning does not occur. Partial melting also serves to create buoyant crust, which takes time to cool. Thick crust produced at high mantle potential temperatures is hard to subduct. I discuss evidence from the present Earth, relating to each phenomenon starting with subduction.

The existence of a buoyant crust limits the efficacy of plate tectonics at times when the interior of a planet is hot. In particular, modern style plate tectonics could not transfer the heat supplied by radioactivity at times when the interior of the early Earth was much hotter than at present [Davies, 1993b, Sleep, 1994, Vlaar et al., 1994]. The thickness of magma released by partial melting, and hence the thickness of oceanic crust, scales to the temperature of the source region. Crudely, a temperature increase of 15 K gives an additional kilometer of oceanic crust [McKenzie and Bickle, 1988]. The oceanic crust and some of the mantle beneath it need to cool for subduction to occur. This cooling time scales to the square of the crustal thickness. Crust younger than 5-10 m.y. appears to be difficult to subduct on the modern Earth. For example, consider a time in the past when the mantle was 300 K hotter than at present. The oceanic crust was then 20 km thicker than the present present 6 km. The minimum age at subduction is (26/6) squared the present value or 94-188 m.y. For the shorter time the global rate of seafloor spreading for an ocean covered earth would have been 5.4 km²/yr⁻¹, and the global average heat flow would have been 103 mW/m². This is insufficient to balance the heat provided by radioactivity in the Hadean.

The situation appears to be worse for the smaller planets [Warren, 1993; Sleep, 1994]. The thickness of crust produced at a given interior temperature scales inversely to the pressure gradient $p_g$ or, crudely, inversely to $g$. In the above example the conditions to produce 26-km-thick crust on the Earth would produce 68-thick-km crust on Mars, which would be quite difficult to subduct. Current Earth conditions would produce 16-km-thick crust on Mars, which can now be subducted on the Earth.

Scaling relationships for heat flow of plate tectonics limited by subduction of thick crust are easily obtained. The simplest criterion is that a cooling time and hence crustal age at subduction scale to crustal thickness squared and hence $g^{-2}$ at a given potential temperature. The globally averaged heat flow $q$ scales with the inverse of the source root of this age and thus linearly with $g$. From (4) and (6) the equivalent heat flows for a given radioactive composition and a given cooling rate also scale with $g$. In this special case the temperature history in (2) becomes independent of planetary size. More likely, sufficiently thick crust may be impossible to subduct even after it has cooled, and a weak dependence on planetary size exists.

With regard to the demise of plate tectonics at low mantle temperatures, the current mantle of the Earth typically produces 6-km-thick oceanic crust. Global cooling of the mantle by another 50-100 K will preclude significant melting at ridge axes [Sleep, 1994]. Local regions provide partial analogs to the conditions at the eventual demise of seafloor spreading when the ascending mantle is too cold to melt effectively and the flowing mantle is so viscous that spreading rates are quite slow.

Unusually thin oceanic crust now forming above cool upwelling mantle south of Australia provides an example of the local effects of cool upwelling mantle. The remainder of the ridge boundary, however, has normal crust. The cool segments of the ridge axis spread, but the axial zone is partially pinned and extends by faulting.

Ridge axes abandoned during plate boundary reorganizations provide an example of the effects of slow spreading rates on pinning, where normal temperature mantle upwells. The Labrador Sea is a typical case [Osler and Louden, 1995]. Slow spreading followed the plate reorganization. Eventually, spreading ceased altogether even though it was kinematically possible for spreading to continue on both the new (Iceland) and the old ridge axes. During the period of slow spreading, lateral heat conduction cooled the ascending mantle to the point that much less melting and crustal production occurred [Sleep and Barrow, 1997]. The lack of crust melt pinned the axes, which extended by faulting [Osler and Louden, 1995]. Along these lines a minimum full rate of 13.5 mmyr⁻¹ exists for well-defined ridge axes on the Earth [Stoddard, 1992] even though plate reorganizations to create slower axes are not precluded kinematically. The minimum rate scales linearly with crustal thickness and hence linearly with $g$ for a given potential temperature [Sleep, 1994].

To reiterate, the heat flow from plate tectonics is expected to increase with mantle potential temperatures by (8) when the oceanic crust is neither too thick nor too thin. These situations apply on the present Earth except for local regions that do not have a significant global effect. For example, thick plume-affected crust jams subduction zones.

2.3.3. Magma ocean. In analogy to the Moon, a magma ocean is generally considered to have existed on the early Earth [e.g., Abe, 1997]. The Moon, some geochemical evidence from Mars, and magma chambers at modern ridge axes provide some calibration.

Following Abe [1997], a magma ocean would begin on the Earth with molten rock extending down to a depth of several hundred kilometers. After a brief period of time a solid lid would form but frequently founder back into the interior of the magma ocean. The heat flow at the surface would be limited by the rate at which the lid formed and sank. Still later, the interior of the magma ocean was mostly mush. Heat was transferred by solid-state convection from the deeper interior of the planet to the magma ocean, by melt movement and mush convection within the ocean, and by conduction through and overturning of the solid lid. The heat flow from these processes is uncertain. Abe [1997] envisions a slowly convecting magma ocean with a turnover time of around a million years and an average heat flow of 1 Wm⁻², which could persist for around 200 m.y. This exceeded $q_o$ on the early Earth, implying that it cooled. The axial regions of fast ridge axes where a mush-filled chamber exists below a thin magma lens are another calibration. The heat flow from the lens is over 10 Wm⁻², which could persist globally for around 20 m.y.

On the Moon, in contrast, a buoyant lid formed by feldspar flotation formed at the top of the magma chamber and became the upper lunar crust [Warren, 1993]. Whether the chamber was molten or mostly mushy, conduction through this static lid limited heat flow. The upper crust did not founder extensively back into the magma ocean.

Scaling with planetary radius can be done. The thickness of a magma ocean at a given potential temperature scales inversely with pressure or gravity, as does the thickness of a buoyant
feldspar lid that prevents overturn. The transition point between the static-lid magma ocean and the active-lid one must lie somewhere between the Earth and the Moon. Geochemical evidence from Mars indicates that it had an active lid that returned into the mantle. There is no evidence of a thick buoyant lid like on the Moon or of extensive formation of Earth-like continents [Blichert-Toft et al., 1999].

2.4. Mode Transitions

I have briefly reviewed the physics of stagnant-lid convection, plate tectonics, and magma oceans. The fundamental involvement of melting in plate tectonics and magma oceans may allow transitions to occur as geologically sudden jumps between modes. Six transitions are conceivable.

2.4.1. Ridge lock. First consider plate tectonics (Figure 1). I begin with the "ridge lock" transition from plate tectonics to stagnant-lid convection as the planet cools. As discussed in section 2.3.2, the mantle needs to be hot enough for basalt to melt at upwellings at the ridge axes. The partial melt allows the shallow part of the ridge axis to spread easily, as the crustal magma chamber and dike intrusion zone have little strength. For this melting to occur, the mantle adiabat needs only to be hot enough that it intersects the melting curve at shallow depth beneath the ridge axis, where the lithosphere is already thin. Once the mantle adiabat is too cool for much melting to occur on an ascending adiabat, the ridge axis behaves as a strong region, locking the plates and stopping plate tectonics once spreading rates have dropped below a critical value.

Once plate tectonics stops, thick global lithospheric lid forms. Minor amounts of extension, like Valles Marineris on Mars, and compression may persist but transfer globally negligible amounts of heat. Lithospheric thickness differences associated

Figure 1. The demise and restart of plate tectonics are shown schematically. (top) The large region of partial melting in the upwelling mantle which exists beneath ridge axes supplies basalt to the crust allowing spreading to occur easily. The potential temperature needs to be moderately hot for enough melting to occur. (middle) Little melting occurs in the ascending mantle if it is too cold. This lack of crustal melts locks the ridge axis, ending plate tectonics. Once the lithosphere has thickened, a much higher potential temperature is needed for extensive melting at the base of the plates. (bottom) Plate tectonics (or a magma ocean) may restart as the hot mantle material impinges into the lithosphere.
with the previous plates also persist but lose their identities on a timescale comparable to the age of the older oceanic crust at the time of the stoppage.

2.4.2. Trench lock. I now discuss the "trench lock" transition from plate tectonics to a magma ocean as a planetary interior heats up. If the heat flow associated with plate tectonics is less than that produced by radioactivity in (2), the interior of the planet will heat up. This leads to an instability once the interior becomes hot enough that the oceanic crust is thick. As discussed in section 2.3.2, a minimum condition for subduction is that the oceanic crust must freeze to its base. This implies that the maximum allowable spreading rate decreases and that global heat flow decreases as progressively thicker oceanic crust is produced at progressively higher mantle temperatures. Eventually, magma ocean convection results when the thickness of the basal molten layer of the oceanic crust becomes great enough that cool upper crustal blocks can founder.

This process weakly depends on planetary radius because, as noted in section 2.3.2, thick oceanic crust occurs preferentially on small planets, where pressure increases slowly with depth. Conversely, subduction and hence plate tectonics are favored on large planets, where the oceanic crustal thickness at a given potential temperature is smaller. If plate tectonics occurred on Mars, the transition would be between it and the Moon.

2.4.3. Maintenance and demise of stagnant-lid convection. Stagnant-lid convection may evolve into plate tectonics by a mechanical instability associated with the finite strength of the lithosphere [Moresi and Solomonov, 1998] or into plate tectonics or a magma ocean by a melting instability [Reese et al., 1998, 1999]. I consider the melting instabilities resulting in plate tectonics and resulting in magma oceans together, as the outcome depends on whether thick crust formed after the initial overturn will be able to subduct. In both cases the mantle potential temperature needs to be hot enough for significant melting to occur at the base of the lid. This mantle potential temperature is likely to be considerably hotter than that required to maintain plate tectonics where melting occurs at shallow depths beneath ridges.

To obtain scaling times for the demise of stagnant-lid convection, I begin with the lithospheric thickness that is in equilibrium with radioactivity in the mantle,

\[ z_L = kT_L / q_r = kT_L / r_p \rho A, \]

(10)

where \( T_L \) is the temperature difference across the lithosphere. This thickness provides a basic length scale. First, the potential temperature difference between that where plate tectonics ends with melting barely occurring at the surface and when it restarts with melting occurring at the base of the lithosphere is not strongly dependent on planetary size. The difference in melting (potential) temperature over the depth range \( z_L \) is

\[ \Delta T_m = \frac{\partial T_m}{\partial P} \Delta P, \]

(11)

where \( \partial T_m / \partial P \) is the melting point gradient with respect to pressure. \( \Delta P = \rho g z_L \) is the pressure change over the depth range \( z_L \). Applying (3) for \( \tau \), the temperature difference is

\[ \Delta T_m = \frac{\partial T_m}{\partial P} \frac{4\pi \kappa L g \rho T_L}{A}, \]

(12)

which is independent of planetary radius \( r_p \). The pressure change \( \Delta P \) is also independent of planetary radius.

The time for an initially hot lid to freeze downward to its equilibrium thickness and the time for the mantle heat up by radioactive heat generation provide another scaling which is useful for appraising the assumption that the lithospheric thickness and surface heat flow quickly adjust to the internal conditions in the planet. I obtain a generally applicable scaling for stagnant-lid convection by considering events following the demise of plate tectonics. If stagnant-lid convection is initially sluggish enough to ignore in the heat balance, the lid thickens with the square root of time, dimensionally

\[ L = \sqrt{\Delta t} . \]

(13)

The time to form the equilibrium thickness lid is dimensionally,

\[ t_L = z_L^2 / \kappa . \]

(14)

The time for radioactivity to heat the planetary interior \( \Delta T_m \) is from (5)

\[ t_m = \Delta T_m C / A \]

(15)

The dimensionless ratio of the two times is

\[ \Pi = \frac{t_m}{t_L} = \frac{4\pi \kappa L g \rho T_L}{\Delta T_m C / A} . \]

(16)

This ratio is independent of the radioactive heat generation within the planet with the caveat that the times are assumed not to be significantly greater than the timescales on which radioactivity decays.

The expression for the ratio (equation (16)) implies that for large planets, the time for the lithosphere to reach its eventual equilibrium thickness \( t_L \) is short compared to the time for the mantle to heat up \( t_m \). That is, the mantle is still cool when the lithosphere first reaches \( t_L \). As stagnant-lid convection is expected to be sluggish then, the lithosphere continues to thicken. Only after the interior has heated up enough that stagnant-lid convection is vigorous does the lithosphere begin to thin toward its equilibrium thickness. Conversely, the interior of a small planet heats up before the lithosphere can reach its equilibrium thickness.

A calibrated ratio for the planets is obtained by scaling from the current condition of the Earth where the dimensional values of the two times are constrained. The Earth is clearly in the large-planet end of this relationship as the mantle heats up very little (12.5 K for the lower estimate of radioactivity) during the time \( t_L \) of ~100 m.y for its lithosphere to reach an equilibrium thickness ~100 km. Letting \( \Delta T_m \) be 300 K (the change in melting temperature over 100-km depth), the time \( t_m \) is around 2 b.y., and the ratio \( \Pi \) for the Earth is ~24. The \( g^2 \) scaling to other planets is made by noting that the surface gravity scales with \( \rho_r \) and that the melting \( \partial T_m / \partial P \) gradient is not expected to vary much between planets. For Mars and the Moon the temperature contrast across the lithosphere \( T_L \) is similar to that of the Earth. Their gravities are 38 and 16.5% of the Earth’s, implying that the ratio is reduced by factors of 0.14 and 0.027, respectively. The ratio for Venus is 1.32 of that of the Earth because the temperature contrast is about 800/1300 of that of the Earth, while its gravity is 90% of that of the Earth. The ratios \( \Pi \) are then 31.7, 3.4, and 0.65 for Venus, Mars, and the Moon, respectively. This places Venus (safely) and Mars (somewhat) within the large-planet regime and the Moon within the small-planet regime. The latter is not unexpected, as the lunar lithosphere is approximately as thick as that produced by conductive cooling to the surface over the age of the planet and comprises the bulk of the lunar mass.

I now examine the conditions at the base of the lithosphere leading to a melting instability for stagnant-lid convection. The
lithospheric thickness in a large planet is controlled by the balance between convective heat supplied from below and conduction to the surface. The conductive heat flow is

$$q_d = \frac{kT_L}{Z},$$

where $Z$ is the current lithosphere thickness. The heat flow transported by convection depends on the "half-space" viscosity just below the base of the lid from (7). In general, this viscosity is both temperature and pressure dependent. Doin et al. [1997] represent it over the relevant range by a function of the form

$$\eta = \eta_0 \exp \left[ \frac{Z}{D_\eta} \left( \frac{T_L}{T_\eta} \right) \right],$$

where $\eta_0$ is a constant with dimensions of viscosity, $D_\eta$ is the scale depth for viscosity, and $T_\eta$ is the scale temperature. The viscosity varies with depth if viscosity is pressure dependent. The pressure and hence viscosity change over a depth range scaling to the thickness of the equilibrium lithosphere in (11) and (12) is independent of planetary radius (to the first order).

Doin et al. [1997] consider evolution of cratonal lithosphere on the Earth where the underlying potential temperature may be considered as given because most of the heat loss of the planet is associated with plate tectonics. This case also applies instantaneously in the large-planet case where the thickness of the lithosphere changes faster than the potential temperature of the deep interior. They graphically represent the evolution of lithospheric thickness by plotting the quasisteady state heat flow into the base of the lithosphere (here represented by (7)) against lithospheric thickness (Figure 2).

The assumption that the surface heat flow can be represented as $q(T)$ is likely to be satisfied when the convective
heat flow is a weak function of depth, as would occur if the viscosity was not very depth dependent. A single stable equilibrium thickness then exists, and the time to approach this equilibrium scales to this thickness squared (Figure 2, top left). The equilibrium evolves slowly as the temperature of the planetary interior changes.

The time-dependent behavior is shown graphically. At time 1 the planetary interior is cool, and convection is not particularly vigorous. The stable equilibrium is at point A. If the lithosphere is thin, the conductive heat flow out of the lithosphere is greater than the convective heat flow into its base. The net heat loss causes the lithosphere to thicken toward point A. Conversely, if the lithosphere is thicker than point A, convective heat flow exceeds convective heat flow, causing the lithosphere to thin. At time 2 the planetary interior has heated up (Figure 2, top right). The increase in potential temperature increases conductive heat flow at a given lithospheric thickness a little through (17) and the convective heat flow a lot through (7) because viscosity is exponentially dependent on temperature. The stable equilibrium is now at point B, which is significantly shallower than point A.

In general, the lithosphere thickness tracks the intersection of the conductive and convective curves as the mantle heats up (from (1) and (2)), making convection more vigorous. Eventually, an instantaneous equilibrium is obtained when the conductive and convective heat flows both equal the heat flow supplied from below by radioactive heat generation $q_r$. A melting instability will occur before this state is reached if the temperature at the base of the lithosphere becomes large enough. Calculations using more sophisticated rheologies than (18) indicate that the maximum stagnant-lid convective heat flow before extensive melting occurs is $10-20$ mW m$^{-2}$ for Venus and $15-30$ mW m$^{-2}$ for Mars [Reese et al., 1998, 1999]. As they note, this is much less than the heat flow $q_r$ generated by radioactivity early in the solar system.

Doin et al. [1997] also consider the case where viscosity increases rapidly with depth. Then the convective heat flow decreases rapidly with lithospheric thickness. The initial evolution of lithospheric thickness then depends on the starting condition (after the demise of a magma ocean or plate tectonics). If the potential temperature is low enough, the convective heat flow is everywhere less than the conductive heat flow (time 0, Figure 2, bottom left). The lithosphere thickens with time. Once the potential temperature increases some, the situation is more complicated. A stable equilibrium similar to those in the top panel occurs at point A (time 1, Figure 2, bottom left). The equilibrium at point B is unstable. If the lithosphere is thinner than B but thicker than point B, convective heat flow exceeds conductive heat flow and the lithosphere thins toward point A. If the lithosphere is thicker than point B, the conductive heat flow exceeds the convective heat flow and the lithosphere thickens with time.

The evolution becomes simpler with further warming of the planetary interior. The convective heat flow at a given lithospheric depth increases much more than the conductive heat flow, so that it intersects only once with the convective curve at point C (Figure 2, bottom right). This single stable equilibrium behaves similarly to those in the top panels.

An instantaneous equilibrium of interior radioactivity, convective heat flow supplied to the base of the lithosphere, and conductive heat flow to the lithosphere is obtained, given enough time and provided that an instability does not occur first. This can be shown graphically as an increase in interior temperature moves the whole convective curve to the right in Figure 2. This behavior occurs because convective heat flow increases exponentially with temperature, while conductive heat flow increases linearly. Eventually, the conditions at time 2 will prevail if the equilibrium at point A is not reached first (Figure 2, bottom right).

Still, the path to equilibrium depends on the initial conditions. If the lithosphere is thinner than point B at time 1, the instantaneous equilibrium at point A is tracked as in the case of weakly depth dependent convection. If the lithosphere thickness at time 1 is greater than point B, the lithosphere thickens with time. By the time a single stable equilibrium exists as at time 2, the lithosphere may be quite thick, and convection at its base may be sluggish. The heat flow implied by point C may be greater than that $q_r$ in equilibrium with radioactive heat generation. In that case the lithosphere thins toward point C and then tracks the intersection of conductive and convective curves to a lower heat flow in equilibrium with radioactive heat generation. The concept of representing convective heat flow as $q(T)$ is inadequate in this case. The heat balance in the lithosphere, the vigor of convection at its base, and the evolution of temperature in the planetary interior in (1) or (2) need to be considered explicitly.

2.4.4. Demise of magma oceans. The situation for the demise of a magma ocean is that the continuous layer of mush in the ocean ceases to exist (Figure 3). Convection within and below an ongoing magma ocean is efficient. Heat is transferred by solid-state convection within the mush, foundering of the lid into the mush, and melt movements. The upwelling mantle below the ocean is hot enough that partial melting occurs at the top of the ascending mantle. These melts replenish the magma ocean. Conversely, cooled magma ocean material may sink through the mantle into the deep planetary interior. The mantle convection is quite vigorous because the base of the magma ocean acts as a free-slip boundary and cooled residuum and lid material sinking into the mantle help drive the flow.

The magma chamber begins to die when the mantle potential temperature is cool enough that the lid freezes downward at places into the underlying mantle. This greatly retards the underlying mantle convection because its upper boundary becomes coupled to the lid. Plate tectonics starts if the regions of thick lid can subduct. Otherwise, sluggish stagnant-lid convection will start. Smaller planets probably evolve to stagnant lids, while larger ones evolve to plate tectonics. The presence of water in the planetary mantle and in the subducted crust may aid subduction. For this reason, Venus may have evolved to a stagnant lid rather than to plate tectonics [Reese et al., 1998, 1999].

3. Planetary Thermal Evolution Histories

The implications of (2) to the thermal history of a planet are parameterized by letting the heat flow be the function $q(T)$. This approximation is justified if the time for the lithosphere to come into quasi-steady state is short compared with the timescale over which the interior temperature and radioactive heat generation of the planet varied. As shown in section 2.4.3, the criterion is likely to be satisfied for the Earth, Venus, and Mars, but not the Moon.

3.1. Generic Thermal Histories

It is easy to solve (2) for a given parameterization $q(T)$ on a modern computer. In this paper, the objective is to discuss the
effects of various modes of convection and of transitions between them. To do this, it is convenient to graph the function $q(T)$ along with the instantaneous value of the heat flow $q_r$ supplied by radioactivity. The function $q(T)$ may have multiple branches representing different modes of convection for a given value of $T$.

3.1.1. Evolution of a single mode of convection. I begin by analyzing the behavior implied by a continuous monotonic part of the function $q(T)$. Plots of heat flow versus potential temperature and the heat flow from radioactivity heat generation are shown in Figure 4. The heat flow may be either more or less than $q_r$, and the heat flow may either decrease or increase with potential temperature. This implies that four cases exist. I presume that radioactive heat generation decreases slowly with time owing to decay even though it is conceivable that subduction of crustal material could lead to an increase of mantle radioactive heat generation for a period of time. Internal heat generation from tides may also increase with time within the silicate part of a Jovian satellite.

In case A the heat flow increases with potential temperature and is greater than the $q$. From (2), the mantle cools. The heat flow lags $q_r$ as it decreases because $q(T) = q_r + q_r$. Three other situations are possible (Figure 4). Case B has the same sense of slope as in case A, but the heat flow is less than the radioactive heat generation. The planet heats up, increasing the potential temperature and the heat flow, while the radioactive heat generation decreases. The heat flow and heat generation converge with time. Once they have converged, an instantaneous equilibrium of heat flow and radioactivity heat generation occurs and the potential temperature ceases to increase. Thereafter, the radioactive heat generation continues to decrease putting in below the heat flow. The potential temperature then evolves as in case A.

Alternatively, convection might become less efficient with increasing potential temperature, so that the curves slope down to the right. In case C the heat flow is greater than the radioactive heat generation. The planet cools and the heat flow increases moving, the heat flow further from the radioactive heat.

Figure 3. (top) A vigorous magma ocean is shown schematically. The mantle is hot enough that partial melting occurs below the magma ocean. Foundering of the lid, convection within the mush, and melt flow (not shown) transfer heat within the magma ocean. Melt is supplied to the magma ocean from below, and cool parts of the mush and foundered lid sink into the mantle. Vigorous mantle convection sees the magma ocean as a free surface. (below) The magma ocean dies when the mantle becomes cool enough that only a thin layer of mush is present. The mush layer freezes into the mantle, forming a lithosphere at places. This couples the mantle convection to the lid, making it sluggish.
Figure 4. Four possible cases of the slope of the global heat flow versus potential temperature curve and radioactive heat generation show the behavior of the system with time. The current state of the system is a dot, and the radioactive heat generation is a thin horizontal line. The direction of evolution of the system and the decline of radioactive heat generation with time are indicated by arrows.

Figure 5. A generic $q(T)$ diagram shows the continuous change of the global mantle heat flow as a function of potential temperature. The system changes from magma ocean to plate tectonics at point A and from plate tectonic to stagnant-lid convection at point D. Heat flow and potential temperature decrease monotonically as the system tracked the radioactive heat generation $q_r$ on the curve from time 1 to time 2.
generation. In case D the heat flow is less than the radioactive heat generation; the planet heats up, decreasing the heat flow and causing it to diverge from the radioactive heat flow. A case started with equal radioactivity heat generation and heat flow is unstable, as a minor inequality will cause the system to diverge either as in case C or as in case D.

A simple thermal history is implied if the heat flow is a single-valued monotonic function of potential temperature even though different modes of convection occur (Figure 5). Initially, the heat loss from the magma ocean is far greater than the heat supplied by radioactivity. From (2) this implies that the interior of the hypothetical planet cooled with time and tracked along the curve down and to the left from point A, as in case A of Figure 4. With time, the radioactivity contribution to heat flow \( q_r \) decreased. The potential temperature tracked \( q_r \) on the curve but lagged behind it.

3.1.2 Transitions between modes of convection. More complex histories result if \( q(T) \) is a multivalued function with a branch representing each mode of convection. For example, transitions either to or from plate tectonics and stagnant-lid convection are central to discussion of the thermal history of Venus [Turcotte, 1993; Turcotte et al., 1999] and Mars [Sleep, 1994; Nimmo and Stevenson, 2000]. Transitions between stagnant-lid convection and a magma ocean on Venus are examined by Reese et al. [1998, 1999].

I begin with a generic graph of a mode transition (Figure 6). Two physical branches of the \( q(T) \) curve exist. Initially, the planet is on the upper branch, which is above the radioactive heat generation. As in case A (Figure 4), the heat flow evolves along the curve to point A. With a small additional passage of time it evolves to point B, where the branch ends. A slight amount of additional cooling causes the system to jump to point C on physical branch 2. In the diagram the heat flow is less than the radioactivity and evolves toward point D, as in case B of Figure 2.

Some readers will notice that Figure 6 shows a simple case which arises in catastrophe theory for describing the topology of multibranched functions [Thom, 1983]. Thom [1983] applies catastrophe theory to situations where rigorous determination of the branches of a curve (or multidimensional surface) is unlikely, including whether a dog will bite, run, or continue to confront an attacker. I do not use the term "catastrophe" in this paper, as it is preempted with repugnant connotations in the Earth sciences. The topology of a multibranched curve of a single variable (here \( q(T) \)) is obvious enough that sophisticated mathematics are not required.

3.2. Possible Thermal Evolutions of the Terrestrial Planets

I present possible \( q(T) \) diagrams for the thermal evolution of the terrestrial planets, starting with the Earth and working downward in planetary size. No diagram is given for the Moon because the large-planet assumption explicit in the approximation does not apply. The examples are intended to elucidate
reasonable possibilities and are not intended to be exhaustive or rigorous.

3.2.1. The Earth. A possible \(q(T)\) diagram for the Earth is shown in Figure 7. The magma-chamber and stagnant-lid branches are monotonic functions where heat flow increases with potential temperature. The plate tectonic branch has a maximum at point M on the basis of the inference that high potential temperatures retard plate tectonics because subduction is difficult and low potential temperatures retard plate tectonics because spreading is difficult.

Initially, the Earth's interior was quite hot and cooled on the path ABC, as in case A of Figure 4. They details of the evolution of a very hot magma ocean are beyond the scope of this paper. A "freeze up" transition from a magma ocean to plate tectonics occurred from point C to point D. In the Hadean the radioactive heat production exceeded the plate-tectonic heat flow. The interior heated up following the path DMJE, which are cases B and D of Figure 4. At point E a trench lock transition occurred back to a magma ocean. The planet cooled, reaching point C, where the jump to plates repeated.

This cycle of jumps CD and EB repeated until the radioactive heat generation was less than the maximum M that can be supplied by plates. The existence of this maximum complicated the subsequent evolution. If the state of the system was on the magma ocean curve or between points D and M on the plate curve at that time, the potential temperature remained between D and M once the system entered the plate curve (see case B, Figure 4). If not, the system tracked on the plate curve to point E for another jump (see case D, Figure 4) and then returned to the plate curve by a jump at points C and D.

Once the occupied point on the plate curve became greater than the radioactive heat production, the system cooled along the plate curve, as in case A (Figure 4). As shown, the present state of the Earth is approaching ridge lock at point F to stagnant-lid convection at point G. After this jump the interior will heat up. A possible jump back to plates between points H and J will occur in the future, if the heat flow from radioactive heat generation \(q_r\) is still greater than that a point H. Otherwise, point H will never be reached, and the planetary interior will gradually cool by stagnant-lid convection as the radioactive heat generation wanes.

3.2.2. Venus. A possible \(q(T)\) diagram for Venus qualitatively represents a scenario envisioned by following Reese et al. [1998, 1999] (Figure 8). The initial magma chamber cooled along the path ABFC. At point C a freeze up jump from magma ocean to stagnant-lid occurred. The early radioactive heat generation exceeded the stagnant-lid heat flow. The system evolved to E, where a melting instability jump to a magma ocean occurred. The system continues to loop through CDEF and is presently between D and E with a magma ocean last.
being active around 700 Ga. The looping will continue until \( q \) is less than the heat flow at point E.

A hypothetical plate branch is shown in the diagram for Venus. This curve represents a valid state if Venus was somehow started that way. This mode, however, can never be reached if the planet is started as a magma ocean or even as a stagnant-lid object. This feature is intended to illustrate that the starting condition may matter for the subsequent history in some cases, even when the object starts relatively hot.

3.2.3. Mars. The Mars diagram (Figure 9) differs from that of the Earth in that the plate heat flow exceeded the Noachian heat production. The planet cooled on the path ABC, where a freeze up to plates occurred. The system then evolved to a ridge lock jump to a stagnant lid at points E and F. Since then the planet has heated up. The melting instability to a magma ocean at F and B will not occur because the heat flow cannot exceed the radioactive heat production at point G (see case B, Figure 4). The part of the plate curve for higher potential temperatures than that of point D was never occupied unless the planet started there. This branch, however, would have been occupied if the Noachian \( q \) was instead greater than the plate heat flow.

3.3. Heat Flow Also Dependent on Planetary Composition

The approach may be extended to where the global mantle heat flow \( q \) is a function of more than one variable. One feature of formal catastrophe theory is useful for visualizing how do to this. A mathematical branch connecting the ends (points A and E, Figure 6) of the two physical branches exists [Thom, 1983]. If the curve is considered to have a top and bottom surface, it is inverted along the mathematical branch. The mathematical branch can never be occupied. At point B, occupation of the branch would require the interior to heat up when the heat flow was greater than the heat generation. Occupation at point E would require the interior to cool when heat flow is less than heat generation. Rather, the heat flow jumps to point F on physical branch 1. Mathematical branches are omitted on all \( q(T) \) diagrams, as they only add clutter and do not represent any physically describable mode of convection.

Given that the differences between the Earth and Venus have been attributed to the lack of water in the mantle of the latter, the amount of water \( W \) in the mantle is discussed as an example. In general, both \( W \) and \( T \) will evolve with time, and (in this case) the evolution of \( W \) needs to be related to the two variables. On the Earth this would be done by considering degassing at ridge axes, hydration and subduction of oceanic crust, and the partial degassing of the subducted crust at island arcs. Then function \( q(T,W) \) can be graphed as a two-dimensional surface in a three-dimensional \( q,T,W \) space, that is, as a sheet with folds and cusps [Thom, 1983]. The upright surface of the sheet (as with the line in Figure 6) represents physical branches, and the inverted surfaces represent mathematical branches. Branch jumps occur when the system evolves to the nose of a fold where the sheet changes from upright to inverted. Cusps where multivalued branches are present for
some but not all of the range of the two variables are possible. For example, a $T,W$ path of the system might have a jump from magma ocean to plates at one value of $W$ but have a continuous transition back to a magma ocean after $W$ has evolved to a different value.

Returning to the actual planets, the $q(T)$ diagram for $W$ of Venus might resemble Figure 8, and the one for $W$ for the Earth might resemble Figure 7. A $q(T,W)$ history is relevant to Venus, where hydrogen has been lost to space. Plate tectonics may have existed on the early wet planet and even jumped to stagnant lid-convection as the mantle dried. The plate tectonics branch may not exist or be unreachable on the current dry planet.

A restriction on $W$ is that, like $T$, it changes slowly with time. This requirement is satisfied for the present Earth. First, the abundant $^{40}$Ar in the mantle indicates that degassing has been inefficient over the last 3 b.y. [Sleep, 1979; Williams and Pan, 1992; Tajika and Matsui, 1993; Phipps Morgan, 1998]. Viewed in another way, only the part of oceanic lithosphere that is the residuum of partial melting to form mid-ocean ridge basalts (MORB) is effectively degassed beneath ridges. The thickness of this, 56 km [Langmuir et al., 1992], is less that the thickness of the region from which heat is extracted.

Other possible variables come to mind. A variable regarding chemical stratification and composition of the planet and one involving the temperature contrast across the bottom boundary layer are conceivable. The fraction of the surface area covered by buoyant continents is another possibility. The surface temperature, contrasting the Earth and Venus, is yet another. For such a parameterization to be useful, the effect of the variable on heat flow should be significant and at least qualitatively understood.

4. Conclusions

Possible thermal histories of the terrestrial planets are graphically illustrated by plotting the mantle heat flow as a function $q(T)$ of the interior potential temperature. This approximation is justified if the lithosphere is thin compared to the size of the planet. This criterion is easily satisfied by the Earth and Venus and satisfied enough by Mars that the approach is qualitatively useful. It is not satisfied by the Moon, where the lithosphere makes up the bulk of the planet. The function $q(T)$ may be branched, representing magma oceans, plate tectonics, and stagnant-lid convection. Jumps occur when the system evolves to the end of a branch. Physical examples include the demise of a magma ocean to plate tectonics or stagnant-lid convection by freeze up, the demise of stagnant-lid convection to plates or a magma ocean by a melting instability, the demise of plate tectonics to stagnant-lid convection by ridge lock, and the demise of plate tectonics to a magma ocean by trench lock.
The existence of multiple $q(T)$ branches results partly because melting of the mantle is pressure as well as temperature dependent. The rheology of the lithosphere is also important. For example, the persistence of plate tectonics once started can be qualitatively understood and dynamically modeled in regard to a rheology where damage associated with strain weakens the lithosphere so that steady state strain rate weakening occurs at high strain rates associated with plate boundaries [Tackley, 1998, 2000; Bercovici, 1998]. Conversely, plate tectonics is difficult to restart at the expense of a stagnant lid, as intact lithosphere cannot be easily strained enough for damage to cause weakening.

For now, the $q(T)$ diagrams provide a shopping list for phenomenological descriptions of planetary thermal histories. Better understanding of the rheology of the lithospheric and of the underlying mantle would quantify $q(T)$ and its branch jumps. Still, gross features of planetary evolution are illustrated by my qualitative approach.

The potential temperature of stagnant-lid planets, Venus and Mars, may be even higher than the potential temperature of the Earth and increasing at the present. In contrast, the interior of the Earth has been vigorously cooled by plate tectonics.

Cycles through a loop of jumps between branches are possible if the heat flow supplied by radioactivity $q$, lies between the heat flows of the two branches. Such looping between plates and a magma ocean may have occurred on the early Earth, and Venus may continue to loop between stagnant-lid convection and a magma ocean. Venus may have cycled to and from being a stagnant-lid planet [Turcotte, 1993; Turcotte et al., 1999]. Mars may have had more than one episode of plate tectonics although this is not shown in Figure 9.

For planets heated by radioactivity, the number of cycles through a loop is limited by the amount of available heat. Only a few loops are possible in a given planet's history, as the temperature change for a loop is of the order of a few hundred kelvins, independent of planetary size. It would take well over a billion years for this temperature increase to occur by radioactive heating in the immediate future and hundreds of millions of years within the early planets.

Evolution of a planet along various branches of its $q(T)$ curve leaves a geological record. Events before the most recent demise of either plate tectonics or a magma ocean should be evident on the stagnant-lid plates, including whether the transition was gradual or sudden. The early history of the Earth is somewhat preserved on the continents but not yet understood in terms of the mode of global tectonics. Geochemical and isotopic evidence should exist for brief but intense magma ocean episodes which cycled of the mantle through the zone of melting. Right now, there is some evidence that plate tectonics occurred before Mars became a stagnant-lid planet. A strong magnetic field is expected to exist during these episodes where the mantle is cooling rapidly (resulting in a large temperature contrast between the ablationmantle and the core) and to be absent when the mantle is heating up [Nimmo and Stevenson, 2000]. The lithosphere of Venus is now a stagnant lid. Dehydration of the planet, as well as cooling, may have contributed to this condition.

Acknowledgments. A preprint supplied by Francis Nimmo and Dave Stevenson and subsequent discussions redirected my interest in changes in the modes of planetary convection. I thank Louis Moret and an anonymous reviewer for constructive reviews and David Bercovici for discussions on damage rheology and plate tectonics. This research was in part supported by NSF grant EAR-9902079.

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