Tidal heating and convection in Io

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[1] The prodigious heat flux emitted from the surface of Jupiter’s moon Io is produced in the interior of the satellite by viscoelastic dissipation of tidal energy and is generally thought to be brought to the surface by convective motions. New models of Io’s equilibrium thermal state are constructed using self-consistent calculations of tidal heating and convective heat transport. These models show that while a high-temperature convective equilibrium exists, it falls an order of magnitude short of explaining the observed heat flux. Either Io is currently out of thermal equilibrium, or another heat transport mechanism such as melt segregation determines Io’s thermal state.


KEYWORDS: tidal dissipation, mantle convection, Io, Galilean satellites, thermal evolution


1. Introduction

[2] Io’s internal heat production is dominated by tidal dissipation, unlike any other Solar System body. The heat production rate is sufficient to melt portions of Io’s interior, resulting in the most volcanic activity of any solar system body. Io’s surface heat flow is observed to be $1 - 2 \times 10^{14}$ W at present [Veeder et al., 1994; Spencer et al., 2000], which is several orders of magnitude greater than can be explained by radiogenic heating.

[3] Heating by tidal dissipation is caused by deformation in Io’s mantle. This deformation is driven by the time-varying gravitational potential Io experiences during its elliptical orbit around Jupiter. The silicate rocks which make up Io’s mantle behave viscoelastically (the simplest version of which is Maxwell viscoelasticity) and will therefore dissipate the most energy when the period of the forcing (the period of Io’s tides) is nearest the Maxwell time of the material, given by the ratio of the viscosity to the shear modulus. Thus the amount and distribution of heating depends on the variation of shear modulus and viscosity (and to a lesser extent, density) within Io. Since these material properties, particularly viscosity, are functions of temperature, tidal heat generation depends strongly on the temperature in the mantle and lithosphere of Io.

[4] Heat transport by convection in Io’s mantle also depends strongly on the temperature, increasing with temperature as the viscosity drops (Figure 1). A dramatic increase in convective heat flux occurs above the solidus, as the viscosity drops by several orders of magnitude from that of the solid to that of the liquid. For radiogenically heated planets, the monotonic increase of convective efficiency with temperature guarantees that there will be a single, stable equilibrium temperature (Tozer’s [1965] principle) at which radiogenic heat production is balanced by convective heat loss (low-temperature equilibrium in Figure 1). The addition of temperature-dependent tidal heating makes possible another stable equilibrium point where tidal heat input is balanced by convective heat transport [Segatz et al., 1988; Fischer and Spohn, 1990; Wiemhöfer and Spohn, 1995] as shown in Figure 1. That these equilibria are stable, and the one between is not, is evident from the figure: any increase in temperature from one of the stable points leads to heat transport that exceeds heat generation, thus returning the system to the stable point, and vice versa. The mantle temperature at the new stable equilibrium point is above the solidus, and Io is generally thought to be in or near this equilibrium state.

[5] In this report, new models of Io’s equilibrium thermal state are presented, using recently developed convective heat flow parameterizations [Reese et al., 1999; Solomatov and Moresi, 2000] and a self-consistent calculation of tidal heating. The new models presented here demonstrate that Io’s current heat flow is an order of magnitude too large to be consistent with the convective equilibrium state described above. This is in part due to an increase in Io’s estimated heat flux [Spencer et al., 2000], and in part due to improved convective parameterizations that account for temperature-dependent viscosity. Io’s heat transport is therefore either out of equilibrium with heat production, perhaps in an oscillatory state [Ojakangas and Stevenson, 1986], or convection has been replaced by a different, more efficient heat transport mechanism such as magma migration [Moore, 2001].

2. Tidal Heat Production

[6] Given the density, viscosity, and shear modulus as a function of radius within Io, the tidal heating may be calculated by solving the equations of motion in a layered viscoelastic sphere [Moore and Schubert, 2000] subject to
The following, time-dependent gravitational potential [Kaula, 1964]:

\[ \Phi = r^2 w^2 \left\{ -\frac{3}{2} P^0_2(\cos q) \cos \omega t + \frac{1}{4} P^2_2(\cos q) \right. \\
\left. + [3 \cos \omega t \cos 2f + 4 \sin \omega t \sin 2f] \right\} \]  

where \( r \) is radius from the center of Io, \( w \) is the orbital angular frequency \( (6.54 \times 10^{-6} \text{ rad s}^{-1}) \), \( e \) is the orbital eccentricity \( (0.0041) \), \( q \) and \( f \) are the colatitude and longitude with zero longitude at the sub-Jovian point, \( t \) is time since passage through perijove, and \( P^0_2 \) and \( P^2_2 \) are associated Legendre polynomials.

The deformation of Io is solved for given the tidal potential (equation (1)) and the \( r \)-dependent material parameters. The quasi-static equations of motion and Poisson’s equation for the potential are Fourier transformed in time, resulting in:

\[ u_{ij} = 0 \]  
\[ t^{(\theta)}_{ij} = 0; t^{(\theta)}_i = -d_{ij} p^{(\theta)} + m(s)(u_{ij} + u_{ji}) \]  
\[ f^{(\Delta)} = 0 \]  

where each variable is the complex Fourier amplitude which is a function of frequency \( s = iw \), \( u \) is the displacement vector, \( t^{(\theta)} \) is the isopotential stress tensor, \( p^{(\theta)} \) is the isopotential pressure, \( q^{(\Delta)} \) is the local gradient of the gravitational potential \( f^{(\Delta)} \), and the usual index conventions apply. The isopotential \( \theta \) and local \( \Delta \) Lagrangian fields are related to the more conventional material \( \theta \) fields by:

\[ f^{(\Delta)} = f^{(\theta)} + f^\theta_2(u_k - d_k) \]  
\[ f^{(\Delta)} = f^{(\Delta)} + f^\Delta_2 u_k \]

where \( f \) is any field, \( f^\theta_0 \) is the reference (hydrostatic) value of that field, and \( d \) is the displacement of the isotopothermal surface originally associated with the material element.

The complex rigidity \( m(s) \) in equation (3) is \( s \) times the Fourier transform of the Maxwell stress relaxation function:

\[ m(s) = \frac{\sin s}{s + \omega m h} \]  

where the constant \( m \) on the right side is the elastic rigidity and \( h \) is the viscosity. The ratio \( \omega m h \) is the inverse Maxwell time. It is the imaginary part of \( m(s) \) that controls dissipation, because this leads to deformation that is out of phase with the forcing.

Writing equations (2)–(4) in terms of spherical harmonics and restricting the solutions to spheroidal deformations (tides do not excite toroidal modes in the linear, layered system considered here) results in a sixth-order system which has analytical (power-law) solutions in layers with constant material properties [Wolf, 1991]. These solutions are propagated through the layers from the center to the surface where boundary conditions are applied to determine the unknown conditions at the center. Liquid layers are represented by zero shear modulus and have no unique solutions for the displacements [Dahlen, 1974] (inertial motions have been ignored). Once the deformations are known, the dissipation averaged over an orbital period may be calculated at any point in the body.

As a simple example, the tidal heating generated in a two-layer (solid silicate mantle and liquid metallic core) Io is shown in Figure 2 as a function of the viscosity (h) and shear modulus (m) of the mantle. Also shown is a rheological trajectory which gives the viscosity and shear modulus as a function of temperature marked at intervals along the trajectory. The solid rheology employed here is the same as the rheology used by Fischer and Spohn [1990] with the following exceptions. Above the solidus, the viscosity is multiplied by a term of the form \( \exp(-BF) \), where \( B \) is a dimensionless melt fraction coefficient ranging from 10 to 40, and \( f \) is the volume melt fraction. Above the disaggregation point (40% to 60% melt), the Roscoe-Einstein relationship is used to relate the viscosity to the melt fraction: \( h = h_f(1.35f - 0.35 - 3f^2) \), where \( h_f = 1 \times 10^{-7}\text{exp}(\frac{14.10^6}{T}) \) Pa s is the melt viscosity, which applies above the liquidus. The shear modulus follows the rheology of Fischer and Spohn [1990] up to the disaggregation point, above which it is set to a very small value \( (10^{-7} \text{ Pa}) \). The solidus is assumed to be at 1598 K and the liquidus at 1698 K making the critical temperature (1600 K) correspond to 2% melt. Melt fraction increases linearly from the solidus to the liquidus at a rate of 1% per Kelvin.

As the temperature increases, the viscosity decreases and the tidal heating increases, as shown in the inset of Figure 2. At a certain point slightly above the solidus, the shear modulus begins to decrease while the viscosity decreases more rapidly [Berckheuer et al., 1982]. The tidal heating may continue to increase to a peak value and then decrease rapidly. When enough of the solid is melted to disaggregate the grains, the shear modulus drops suddenly.

Figure 1. A cartoon showing the variation in convective heat loss (dashed line), and radiogenic (dash-dotted line) and tidal (solid line) heat production as a function of mantle temperature. The solidus \( T_s \), critical \( T_c \), breakdown \( T_b \), and liquidus \( T_l \) temperatures are shown. \( T_c \) is the temperature at which the shear modulus begins to drop, above \( T_b \) the mixture behaves as a suspension of particles. Equilibria are shown as circles, with the low-temperature, filled circle corresponding to Tozer’s equilibrium.
Figure 2. Contours of tidal heating [W] in a two-layer Io as a function of viscosity (\(\eta\)) and shear modulus (\(q\)) of the mantle. The colored line is a rheological trajectory giving the viscosity and shear modulus as a function of temperature. The tidal heating as a function of temperature along the trajectory is shown in the inset.

3. Convective Heat Loss

[12] The heat generated by tidal dissipation within Io must be carried to the surface and radiated to space if Io is to achieve thermal equilibrium. In modeling this process, it is typically assumed that convective motions in Io’s mantle are responsible for delivering the heat to the surface. Convection in spherical shells has been studied extensively, and parameterizations including the effects of temperature-dependent viscosity and spherical geometry have been derived and fit to the results of numerical calculations [Reese et al., 1999; Solomatov and Moresi, 2000].

[13] For fluids with strongly temperature dependent viscosity, a portion of the upper thermal boundary layer stagnates and transports heat only by conduction. An actively convecting boundary layer exists between the stagnant lid above and the well-mixed interior below. The convection in the interior is essentially isoviscous and is driven by a rheological temperature scale that depends on the temperature dependence of the viscosity \(h\):

\[
\Delta T_{th} = \left| \frac{h}{dn = dT} \right|.
\]

[14] For a strictly temperature dependent viscosity (pressure dependence should be weak in Io) \(\Delta T_{th} = RT^2/E\) ~50 K where \(R\) is the gas constant and \(E\) is the activation energy (300–500 kJ mol\(^{-1}\)).

[15] Since only part of the shell (below the lid) is convecting, we define a Rayleigh number \(Ra_{th}\) for the shell as follows:

\[
Ra_{th} = \frac{a_r \Delta T_{th} (r_i - r_b)}{kh(T_i)}
\]

where \(a_r\) is the thermal expansivity \(3 \times 10^{-5} \text{ K}^{-1}\), \(r\) is the density of the mantle \(3300 \text{ kg m}^{-3}\), \(g\) is the gravitational acceleration \(1.8 \text{ m s}^{-2}\), \(r_i\) and \(r_b\) are the radii of the bottom of the lid and bottom of the shell, respectively, \(k\) is the thermal diffusivity \(10^{-6} \text{ m}^2 \text{s}^{-1}\), and the viscosity \(h\) is evaluated at the mean temperature of the well-mixed interior \(T_i\). The heat flux transported convectively by the shell to the base of the lid is therefore:

\[
F_{conv} = a_r k \frac{\Delta T_{th}}{(r_i - r_b)} Ra_{th}^3
\]

where \(k\) is the thermal conductivity \(4 \text{ W m}^{-1} \text{ K}^{-1}\), and \(a_r\) \((1.3–2.5)\) and \(b\) \((1/3)\) are constants [Solomatov and Moresi, 2000]. At equilibrium, this flux must balance the production of heat in the shell:

\[
F_{t tidal} = \frac{\tau H r_i}{3 (1 - r_b^2 \pi r_i^4)}
\]

where \(H\) is the volumetric tidal heat production rate, which is here assumed to be constant throughout the shell.
and the flux into the base of the lid $F_{\text{conv}}$. This gives an expression for the temperature at the base of the lid:

$$T_i = T_i - a_{\text{th}} \Delta T_{\text{th}}$$

$$= T_i + \frac{rH}{6k} \left( r_i^2 - r_f^2 \right) + \left( \frac{F_{\text{conv}} r_i^2}{k} - \frac{rH r_i^2}{3k} \right) \left( \frac{1}{r_f} - \frac{1}{r_i} \right)$$

(12)

where $r_i$ is the radius of the top of the shell (Io’s radius) and $T_i$ has been equated to the temperature of the interior $T_f$ minus the temperature drop $a_{\text{th}} \Delta T_{\text{th}}$ across the active boundary layer ($a_{\text{th}}$ is a constant with a value of 2.4 for the Newtonian rheology used here).

(17) Given $T_i$, equations (8)–(12) may be solved for the thickness of the lid $(r_f-r_i)$ and the heat production rate $H$ which are in equilibrium with the convective heat transport (see Reese et al. [1999], but note that in their equation (15), the term $(r_f-r_i)^{S_b}$ should be replaced by $(r_f-r_i)^{S_b}$). Finally, the thickness of the active boundary layer $d_{\text{th}}$ is given by:

$$d_{\text{th}} = \frac{k a_{\text{th}} \Delta T_{\text{th}}}{F_{\text{conv}}}$$

(13)

allowing a complete description of the temperature in the interior of the satellite, as shown in Figure 3.

4. Coupled Tidal-Convective Model

(18) Given a temperature profile such as that shown in Figure 3, the two-layer calculations of Figure 2 can be improved by determining the radially dependent viscosity and shear modulus. Using a multi-layered model (hundreds of layers are possible), the tidal heating in Io’s mantle consistent with the convective temperature structure may be calculated, as shown in Figure 4 for two different rheologies.

Figure 3. Example temperature profile implied by the convective parameterization used in this study. Temperatures in the conductive lid are from a solution of the conduction equation using the heat flux from the convecting interior as a boundary condition. The heat flux and thickness of the lid are calculated given the temperature of the well mixed interior (1500 K in this example).

[16] The thickness of the lid is determined from the solution of the conduction equation given the internal heating rate in the lid (assumed to be equal to the heating rate in the convecting layer $H$), the surface temperature $T_s$.

Figure 4. a) Two different rheologies (thick versus thin lines) bounding the behavior of molten rocks leading to the different tidal heating and convective heat flux curves in b). Equilibrium states exist where the tidal heating and convective heat flux are equal. Despite the different rheologies and different equilibrium temperatures, the equilibrium heat fluxes are both about an order of magnitude below Io’s observed heat flux (gray band).
[19] The solid lines in Figure 4b show the tidal dissipation as functions of temperature for two different rheologies given by the corresponding thick and thin lines in Figure 4a. The rheologies shown bound the range of behaviors measured in partially molten rock [Renner et al., 2000], with the thin line showing the rheology used to calculate the trajectory in Figure 2. The disaggregation point varies from 40% melt (thick lines) to 60% melt (thin lines), and the dependence of viscosity on melt fraction (F) ranges from strong (exp(−40f), thick lines) to weak (exp(−10f), thin lines). The shear modulus (dotted lines in Figure 4a) is independent of temperature below a critical temperature (1600 K) corresponding to a few percent melt [Berckhemer et al., 1982], above which it follows an Arrhenius law with an activation temperature of 4 × 10^4 K. At temperatures above disaggregation, the shear modulus drops rapidly toward a very small value (10^−7 Pa).

[20] Equilibrium points exist where the tidal heating and convective heat flux curves intersect. The equilibria of numerous intermediate models plot in the small gray zone in Figure 4b. The equilibrium temperatures predicted vary considerably, with the very stiff rheology (thin line) resulting in an equilibrium just above the disaggregation point, and the very weak rheology (thick line) resulting in an equilibrium much nearer the solids. The heat fluxes predicted by these models are very consistent; however, they are nearly an order of magnitude too low to explain Io’s observed heat flux (gray band in Figure 4b). This is a very robust result, arising from the nearly equal and opposite dependence of tidal heating and convective heat transport on viscosity. Lowering the viscosity causes the tidal heating to decrease and the convective heat flux to increase by nearly equal amounts, leading to a change in the equilibrium temperature, but leaving the equilibrium heat flux constant.

5. Discussion

[21] The inability of the convective equilibrium state to match the observed heat flux can be looked at in two ways: either the viscosity required to transport 10^{14} W by convection is too low to produce it by dissipation, or the viscosity required to produce that much heat is too high to remove it by convective motions. The combined constraints of observed heat-flow and silicate rock rheology do not yield a self-consistent equilibrium solution.

[22] This dilemma can be resolved by abandoning the assumption of equilibrium, or by proposing another heat transport mechanism that is efficient enough to carry 10^{14} W and does not depend on viscosity in the same way that convective heat transport does. Although thermal-structural states which oscillate about the high-temperature equilibrium have been proposed for Io [Ojakangas and Stevenson, 1986; Fischer and Spohn, 1990; Wiebruch and Spohn, 1995], such states would never be reached if a different heat transport mechanism took over near the solids where the tidal dissipation matches the heat flow. Melt segregation in Io’s mantle is a heat transport mechanism that plausibly meets the requirements [Moore, 2001; Monnereau and Dubuffet, 2002] and is furthermore evidenced by the observations of silicate volcanism on Io’s surface [McEwen et al., 2000]. Io may therefore be an interesting laboratory for studying heat transport processes that were important early in the evolution of the terrestrial planets, when melt segregation was an important heat transport mechanism. Due to the extreme efficiency of heat transport by melt, however, this phase of planetary evolution is necessarily short in planets without strong tidal heating.

[23] It should not be surprising that much of Io’s heat is transported by melt, since even if we ignore what we know about tidal heat generation, the convective models cannot transport 10^{14} W with any reasonable solid-state viscosity. The high temperatures and large melt fractions implied by this imply at least some melt-segregation and volcanism. Indeed, keeping a 50–50 melt-solid suspension well mixed is a difficult problem. Also, the effects of a pressure-dependent solidus are not easily included in convective parameterizations, though attempts have been made [Reese et al., 1998; Monnereau and Dubuffet, 2002].

[24] Since melt segregation is a rapid process compared with the timescales for orbital evolution, oscillatory states are not possible, and the observed heat flux from Io’s surface will be in equilibrium with tidal dissipation. This conclusion may be tested by precise astrometric determination of the rate of change of Io’s orbital frequency, although available measurements are not yet up to the task [Kaas et al., 1999].

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References


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